

Exercise 24

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = 4 + 8x - 5x^2$$

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[4 + 8(x+h) - 5(x+h)^2] - (4 + 8x - 5x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[4 + 8(x+h) - 5(x^2 + 2xh + h^2)] - 4 - 8x + 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{(4 + 8x + 8h - 5x^2 - 10xh - 5h^2) - 4 - 8x + 5x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{8h - 10xh - 5h^2}{h} \\ &= \lim_{h \rightarrow 0} (8 - 10x - 5h) \\ &= 8 - 10x \end{aligned}$$

The domain of $f(x)$ is $\{x \mid -\infty < x < \infty\}$, and the domain of $f'(x)$ is $\{x \mid -\infty < x < \infty\}$. $f(x)$ and $f'(x)$ are polynomials, so any number can be plugged into them.